Student Workbook

Nonlinear Systems and Inequality Regions

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How do you find the intersection points of a line and a curve (or two curves), and how do you graph the solution region of a system of nonlinear inequalities?

Lesson Overview

A nonlinear system contains at least one equation that is not linear — such as a parabola, circle, or hyperbola. Unlike linear systems, nonlinear systems can have 0, 1, 2, or more intersection points.

Substitute / Eliminate → Solve → Graph the Region

Substit.	Solve one equation for a variable, then substitute into the other equation.
Elimin.	Add or subtract equations to eliminate a variable — works well when both equations contain x^2 or y^2 terms.
Inequal.	Graph each boundary curve (solid for ≤/≥, dashed for </>), shade each region, then find the intersection of all shaded regions.
Test Pt	Substitute a point (usually the origin) into the original inequality to determine which side to shade.
Types	Common nonlinear systems: line & parabola, line & circle, two parabolas, parabola & circle.

Worked Examples

EXAMPLE 1

Solve the system: $y = x^2$ and $y = x + 2$.

- Set the expressions equal: $x^2 = x + 2$
- Rearrange: $x^2 - x - 2 = 0$. Factor: $(x - 2)(x + 1) = 0$
- Solve: $x = 2$ or $x = -1$. Find y: when $x=2$, $y=4$; when $x=-1$, $y=1$

Answer: (2, 4) and (−1, 1)

EXAMPLE 2

Solve the system: $x^2 + y^2 = 25$ and $y = x + 1$.

- Substitute $y=x+1$ into the circle: $x^2 + (x+1)^2 = 25$
- Expand: $x^2 + x^2 + 2x + 1 = 25 \rightarrow 2x^2 + 2x - 24 = 0 \rightarrow x^2 + x - 12 = 0$
- Factor: $(x+4)(x-3) = 0 \rightarrow x=-4, y=-3$; or $x=3, y=4$

Answer: (3, 4) and (−4, −3)

EXAMPLE 3

Solve the system: $x^2 + y^2 = 10$ and $x^2 - y^2 = 2$.

- Add the two equations: $2x^2 = 12 \rightarrow x^2 = 6 \rightarrow x = \pm\sqrt{6}$
- Substitute back: $y^2 = 10 - 6 = 4 \rightarrow y = \pm 2$
- All combinations of signs are valid solutions

Answer: $(\sqrt{6}, 2), (\sqrt{6}, -2), (-\sqrt{6}, 2), (-\sqrt{6}, -2)$

EXAMPLE 4

Graph the system of inequalities: $y \geq x^2$ and $y \leq 4$.

- Graph $y=x^2$ (solid — $\geq$ is non-strict) and $y=4$ (solid horizontal line — $\leq$ is non-strict)
- Shade above the parabola and below the line $y=4$
- Check (0,2): $2\geq0$ ✓ and $2\leq4$ ✓ — the origin's neighbor is in the region

Answer: Region between $y = x^2$ and $y = 4$, for $-2 \leq x \leq 2$

EXAMPLE 5

Solve the system: $y = x^2 - 3$ and $y = 2x$.

- Set equal: $x^2 - 3 = 2x \rightarrow x^2 - 2x - 3 = 0$
- Factor: $(x - 3)(x + 1) = 0$
- $x = 3 \rightarrow y = 6$; $x = -1 \rightarrow y = -2$

Answer: (3, 6) and (−1, −2)

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Solve: $y = x^2 - 4$ and $y = 2x - 1$.

Hint: Set the expressions equal and solve the resulting quadratic.

2 Solve: $x^2 + y^2 = 13$ and $x + y = 5$.

Hint: Solve the linear equation for y, then substitute into the circle equation.

3 Solve: $y = x^2$ and $y = -x^2 + 8$.

Hint: Set the two expressions equal — both equal y.

4 Determine the number of solutions: $y = x^2 + 1$ and $y = x - 1$.

Hint: Set equal and check the discriminant of the resulting quadratic.

5 Graph the system: $y \le x^2$ and $y \ge -x + 2$.

Hint: Graph each boundary, test a point in each region, shade the intersection.

Key Vocabulary

Nonlinear System	A system where at least one equation is not linear — it may contain squared terms, products of variables, or other nonlinear expressions.
Substitution	Solving one equation for a variable and substituting that expression into the other equation to reduce the system to one equation in one variable.
Elimination	Adding or subtracting equations to eliminate a variable. Especially useful when both equations contain squared terms (x^2 or y^2).
Solution Region	The set of all points (x, y) that satisfy every inequality in the system — the intersection of all individual shaded regions.
Boundary Curve	The curve (line, parabola, circle, etc.) forming the edge of a solution region. Solid for ≤ or ≥; dashed for < or >.
Test Point	A point substituted into an inequality to determine which side of the boundary to shade. The origin is most convenient when it isn't on the boundary.

Quick Check Quiz

Circle the letter of the correct answer for each question.

1 **How many solutions can a line-parabola system have?** Multiple Choice

A Exactly 1
B Exactly 2
C 0, 1, or 2
D Always 2

2 **Solve: $y = x^2$ and $y = 4$.** Multiple Choice

A (2, 4) only
B (−2, 4) only
C (2, 4) and (−2, 4)
D No solution

3 **For the system $y \le x + 3$ and $y \ge x^2 - 1$, the solution region is:** Multiple Choice

A Above both curves
B Below both curves
C Between the line and parabola
D Outside both curves

4 **Solve: $x^2 + y^2 = 25$ and $x = 3$.** Multiple Choice

A (3, 4) only
B (3, −4) only
C (3, 4) and (3, −4)
D No solution

5 **The boundary line $y = 2x + 1$ is dashed when the inequality is:** Multiple Choice

A $y \le 2x + 1$
B $y \ge 2x + 1$
C $y < 2x + 1$
D $y = 2x + 1$

Common Mistakes to Avoid

✗ Only finding one intersection point when two exist.

✓ Always solve the full quadratic. Factor or use the quadratic formula to find ALL roots — each root gives an intersection point.

✗ Forgetting to find the y-coordinate after finding x.

✓ A solution is an ordered pair (x, y). After finding x, substitute back into either original equation to find y.

✗ Shading the wrong region for an inequality.

✓ Always test a point (like the origin) in the original inequality. If it satisfies the inequality, shade that side.

✗ Using a solid boundary for a strict inequality (< or >).

✓ Strict inequalities (< or >) use a dashed boundary. Non-strict (≤ or ≥) use a solid boundary.

Math Tips

★ For line-parabola systems, substitution is almost always the best method — substitute the linear equation into the quadratic.

★ For two-circle or two-parabola systems, elimination often works better — subtract the equations to cancel the squared terms.

★ The discriminant $b^2 - 4ac$ tells you how many solutions a line-parabola system has: positive → 2 solutions, zero → 1 solution, negative → 0 solutions.

★ When graphing a system of inequalities, shade each region separately first, then identify the overlap.

★ The origin (0, 0) is the easiest test point — unless the boundary passes through the origin, in which case choose (1, 0) or (0, 1).